2018-08-29-1

* No Class
Friday November 16, 2018

* Sections 2.1 – 2.3 in textbook

Last time:

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

$V_{RMS} = \frac{V_m}{\sqrt{2}}$ $\quad I_{RMS} = \frac{I_m}{\sqrt{2}}$

$\bar{V} = V_{RMS} \angle \theta_v$

$\bar{I} = I_{RMS} \angle \theta_i$

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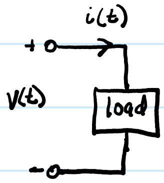
* Today: why use RMS values instead of peak values?
What is complex power?



* Single phase: 1 voltage and 1 current

Instantaneous power: $p(t) = v(t)\,i(t)$

let $v(t) = V_m \cos(\omega t + \theta_v)$
$i(t) = I_m \cos(\omega t + \theta_i)$

$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$

Recall: $\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$

$\Rightarrow p(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)\right]$

* $\cos(2\omega t + \theta_v + \theta_i)$ term causes pulses in power
$\Rightarrow$ pulses in torque for motors and generators
BAD!!

$\cos(2\omega t + \theta_v + \theta_i) = \cos(2\omega t + 2\theta_i + \theta_v - \theta_i) = \cos(2(\omega t + \theta_i) + (\theta_v - \theta_i))$

$= \cos(2(\omega t + \theta_i))\cos(\theta_v - \theta_i) - \sin(2(\omega t + \theta_i))\sin(\theta_v - \theta_i)$

$$\boxed{p(t) = \frac{V_m I_m}{2}\left[(1 + \cos(2\omega t + 2\theta_i))\cos(\theta_v - \theta_i)\right] - \frac{V_m I_m}{2}\sin(2\omega t + 2\theta_i)\sin(\theta_v - \theta_i)}$$

2018-08-29-2

Recall: $V_{RMS} = \frac{V_m}{\sqrt{2}}$ $\quad I_{RMS} = \frac{I_m}{\sqrt{2}}$

$$\frac{V_m I_m}{2} = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) = V_{RMS} I_{RMS}$$

$$p(t) = V_{RMS} I_{RMS}\left[(1+\cos(2\omega t + 2\theta_i))\cos(\theta_v - \theta_i) - \sin(2\omega t + 2\theta_i)\sin(\theta_v - \theta_i)\right]$$

* Using RMS values simplifies the power calculation. No longer have $\frac{1}{2}$ front factor.

* Leads to the question: what is the average power P over a cycle?

$$P = \frac{1}{T}\int_0^T p(t)\,dt \Rightarrow P = \frac{1}{T}\int_0^T V_{RMS} I_{RMS}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + 2\theta_i)\cos(\theta_v - \theta_i) - \sin(2\omega t + 2\theta_i)\sin(\theta_v - \theta_i)\right]dt$$

* The integral of a sinusoid over a multiple of its period is 0

$$\Rightarrow P = \frac{V_{RMS} I_{RMS}}{T}\cos(\theta_v - \theta_i)T \Rightarrow \boxed{P = V_{RMS} I_{RMS}\cos(\theta_v - \theta_i)}$$

We can use this to rewrite the instantaneous power as

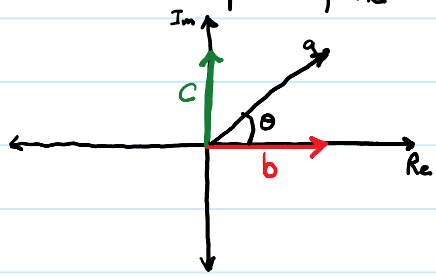
$$p(t) = P(1+\cos(2\omega t + 2\theta_i)) - V_{RMS} I_{RMS}\sin(\theta_v - \theta_i)\sin(2\omega t + 2\theta_i)$$

* Now, define the Reactive Power Q

$$Q = V_{RMS} I_{RMS}\sin(\theta_v - \theta_i)$$

$$\boxed{\begin{aligned} p(t) &= P(1+\cos(2\omega t + 2\theta_i)) - Q\sin(2\omega t + 2\theta_i) \\ P &= V_{RMS} I_{RMS}\cos(\theta_v - \theta_i) \\ Q &= V_{RMS} I_{RMS}\sin(\theta_v - \theta_i) \end{aligned}}$$

Recall the complex plane



$b = Re\{ae^{j\theta}\} = a\cos(\theta)$

$c = Im\{ae^{j\theta}\} \quad a\sin(\theta)$

$P = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) = Re\{V_{RMS} I_{RMS} e^{j(\theta_v - \theta_i)}\}$

$Q = V_{RMS} I_{RMS} \sin(\theta_v - \theta_i) = Im\{V_{RMS} I_{RMS} e^{j(\theta_v - \theta_i)}\}$

$P = Re\{V_{RMS} e^{j\theta_v} I_{RMS} e^{-j\theta_i}\} = Re\{(V_{RMS}\angle\theta_v)(I_{RMS}\angle -\theta_i)\} = Re\{\bar{V}\bar{I}^*\}$

$Q = Im\{V_{RMS} e^{j\theta_v} I_{RMS} e^{-j\theta_i}\} = Im\{(V_{RMS}\angle\theta_v)(I_{RMS}\angle -\theta_i)\} = Im\{\bar{V}\bar{I}^*\}$

<u>Note:</u> $\bar{I}^*$ means the complex conjugate of $\bar{I}$. Switch the sign of the imaginary terms to compute.

So, P and Q are the real and imaginary parts of some phasor $\bar{V}\bar{I}^*$.

This is called the complex power $\bar{S}$

$$\boxed{\bar{S} = \bar{V}\bar{I}^* = V_{RMS} I_{RMS} \angle \theta_v - \theta_i = P + jQ}$$

<u>Units:</u>

$\bar{S}$: VA

P: W

Q: VAR

Different units to emphasize different meanings

<u>Other Nomenclature</u>

$|\bar{S}| = S = VI$: apparent power

$\theta = \theta_v - \theta_i$: power factor angle

$\cos(\theta) = \cos(\theta_v - \theta_i)$: Power factor

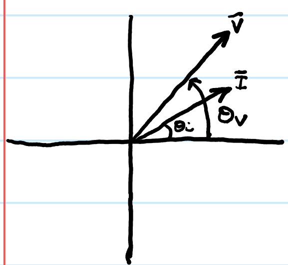
2018-08-29-4

Recall $\cos(\theta)$ is an even function, meaning that

$\cos(\theta) = \cos(-\theta)$.

How to distinguish between the two for the PF?

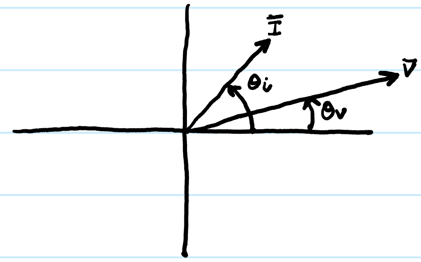
$\theta = \theta_v - \theta_i > 0$



Current lags the voltage

$\Rightarrow$ lagging PF

$\theta = \theta_v - \theta_i < 0$



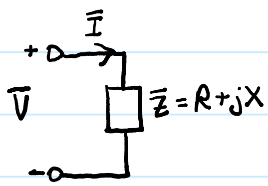
Current leads the voltage

$\Rightarrow$ leading PF

* Talk about current lagging or leading the Voltage

$\bar{S} = \bar{V}\bar{I}^* = V_{RMS} I_{RMS} \angle \theta_v - \theta_i = P + jQ$

Another way to view complex power:



* Note: $\bar{Z}$ is a complex number, but <u>not</u> a phasor. No time dependence

$\bar{V} = \bar{Z}\bar{I}$

$\bar{S} = \bar{V}\bar{I}^* = (\bar{Z}\bar{I})\bar{I}^* = \bar{Z}(\bar{I}\bar{I}^*) = \bar{Z}(I_{RMS}^2) = \bar{Z} I^2$

$\bar{S} = I^2 \bar{Z}$

$Z = R + jX$

$\boxed{\bar{S} = I^2 R + j I^2 X = P + jQ}$

P: power dissipated by resistors

Q: power dissipated by reactances (hence the name reactive power)

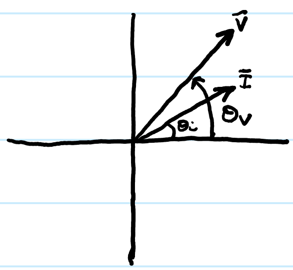
* Note: when calculating an impedence $\vec{Z} = R + jX = Z\angle\theta_Z$
the power factor angle $\theta = \theta_Z$

$\bar{I} = \dfrac{V\angle\theta_v}{Z\angle\theta_Z} = \dfrac{V}{Z}\angle\theta_v - \theta_Z$ $\qquad \bar{S} = S\angle\theta_v - \theta_i \Rightarrow \bar{S} = S\angle\theta_v - (\theta_v - \theta_Z)$

$$\boxed{\bar{S} = S\angle\theta_Z}$$

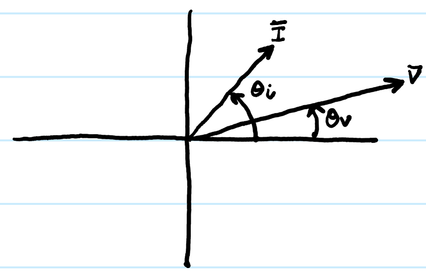
* We can use this other method of viewing the complex power to understand the PF:

$\theta = \theta_v - \theta_i > 0$



Current lags the voltage
$\Rightarrow$ lagging PF

$\theta = \theta_v - \theta_i < 0$



Current leads the voltage
$\Rightarrow$ leading PF

Lagging PF: $\theta > 0 \Rightarrow Q > 0$ (X is like an inductor)
Leading PF: $\theta < 0 \Rightarrow Q < 0$ (X is like an capacitor)

$Q = I^2 X$

Inductor: $X = \omega L \Rightarrow Q > 0$

Capacitor: $X = -\frac{1}{\omega C} \Rightarrow Q < 0$

2018-08-29-6

Ex)

$v(t) = \sqrt{2}(10)\cos(\omega t + 30^\circ) \Rightarrow \bar{V} = 10\angle 30^\circ \text{ V}$

$i(t) = \sqrt{2}(5)\cos(\omega t - 20^\circ) \Rightarrow \bar{I} = 5\angle 20^\circ \text{ A}$

Find: a) The complex power $\bar{S}$

b) The power factor

a) $\bar{S} = \bar{V}\bar{I}^* = (10\angle 30^\circ \text{V})(5\angle 20^\circ \text{A})$

$\boxed{\bar{S} = 50\angle 50^\circ \text{ VA}}$

$\boxed{\bar{S} = 32.14 + j38.30 \text{ VA}}$

b) $PF = \cos(\theta) = \cos(50^\circ)$

$\boxed{PF = 0.643 \text{ lagging}}$

Ex)

$v(t) = \sqrt{2}(10)\cos(\omega t + 10^\circ)$

$i(t) = \sqrt{2}(5)\sin(\omega t + 130^\circ)$

Find: a) Complex power

b) PF

a) $\sin(\omega t + 130^\circ) = \cos(\omega t + 130^\circ - 90^\circ) = \cos(\omega t + 40^\circ)$

$\bar{V} = 10\angle 10^\circ$

$\bar{I} = 5\angle 40^\circ$

$\bar{S} = \bar{V}\bar{I}^* = (10\angle 10^\circ \text{ V})(5\angle -40 \text{ A})$

$\boxed{\bar{S} = 50\angle -30^\circ \text{ VA}}$

$\boxed{\bar{S} = 43.30 - j25 \text{ VA}}$

b) $PF = \cos(-30^\circ)$

$\boxed{PF = 0.866 \text{ lead}}$

Ex)

$v(t) = \sqrt{2}(10)\cos(\omega t + 30^\circ) = \bar{V} = 10\angle 30^\circ \text{ V}$

$i(t) = \sqrt{2}(5)\cos(\omega t - 90^\circ) = \bar{I} = 5\angle -90^\circ \text{ V}$

$\bar{S} = \bar{V}\bar{I}^* \Rightarrow \boxed{\bar{S} = 50\angle 120^\circ \text{ VA}}$

$\boxed{\bar{S} = -25 + j43.3 \text{ VA}}$

* Notes: 1) $P < 0$ means power is delivered by network rather than consumed

2) PF only defined for $|\theta| < 90^\circ$, so we can't define a PF here.